Non-Markovian effect on the geometric phase of a dissipative qubit

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We study the geometric phase of a two-level atom coupled to an environment with Lorentzian spectral density. The non-Markovian effect on the geometric phase is explored analytically and numerically. In the weak coupling limit the lowest-order correction to the geometric phase is derived analytically and the general case is calculated numerically. It is found that the correction to the geometric phase is significantly large if the spectral width is small and in this case the non-Markovian dynamics has a significant impact to the geometric phase. When the spectral width increases, the correction to the geometric phase becomes negligible, which shows the robustness of the geometric phase to the environmental white noises. The result is significant to the quantum information processing based on the geometric phase.

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I. INTRODUCTION

The concept of geometric phase (GP) in quantum system was originally introduced by Berry [1] when he studied the dynamics of a closed quantum system which undergoes an adiabatic cyclic evolution. He found that besides the usual dynamical phase, the system also acquires an additional phase which only depends on the geometry of the path traversed by the system during its adiabatic evolution. Since then this important notion has attracted much attention [2]. It has been generalized in various aspects, e.g. the GP for non-adiabatic evolution [3] and for noncyclic evolution [4]. The GP has been observed experimentally in optical [5], NMR [6, 7], and superconducting electronic circuit experiments [8, 9].

Recently, the renewed interest in the investigation of GP comes from the application of GP to implement the logic gates in quantum computation [10]. The purely geometric nature of the phase makes such computation intrinsically fault-tolerant and robust against certain types of classical fluctuation noise [11–16]. However, any realistic quantum system is inevitably coupled to its surrounding environment, which would result in the loss of quantum coherence (i.e. the decoherence) of the quantum system itself and hence limit the implementation of the geometric quantum computation. Therefore, the study of the GP in open quantum systems becomes an important issue. For the GP of mixed states in open systems, Uhlmann was the first to make the attempt to define the mixed-state GP via state purification [17]. Sjöqvist *et al.* proposed an alternative definition for the nondegenerate mixed-state density matrix under unitary evolution based on the interferometry [18]. This definition was further generalized to degenerate mixed state by Singh *et al.* [19] and to the nonunitary evolution by Tong *et al.* [20] using the kinematic approach. The GP associated with such a nonunitary evolution is defined as [20]

$$\Phi_g = \arg\Big\{ \sum_k \sqrt{\varepsilon_k(0)\varepsilon_k(T)}\, \langle\psi_k(0)|\,\psi_k(T)\rangle \times e^{-\int_0^T dt \langle\psi_k|\frac{\partial}{\partial t}|\psi_k\rangle}\Big\}, \tag{1}$$

where $\varepsilon_k(t)$ and $|\psi_k\rangle$, respectively, are the eigenvalues and the eigenstates of the reduced density matrix of the quantum system, and T is the time after the system completes a cyclic evolution when it is isolated from the environment. Taking the environment into account, the system no longer undergoes a cyclic evolution. Here a quasi-cyclic process with $T = 2\pi/\omega_0$, where ω_0 is the frequency of the system, is considered in Eq. (1). Wang *et al.* defined a mixed-state GP in the context of Pantcharatnam formula [21] via mapping the density matrix to a nonunit vector ray in complex projective Hilbert space [22]. The mixed-state GP has been observed in NMR system [23, 24].

Because the environment induced decoherence would affect the performance of the quantum computation using GP, the study of environment effects on the GP is highly desired. Many works along this line have been performed within Markovian approximation [25–30], which is valid only when the interaction between the system and the environment is very weak and the environmental correlation time is very small. However, in many quantum information experiments, these conditions are not completely satisfied. For example, in cavity QED experiment the imperfection of the cavity mirrors makes the cavity field having a Lorentzian spectrum expansion, which acts as an environment, would exert a strong non-Markovian effect on the atom in it [31]. There are also some works on the non-Markovian effect on the GP in dephasing environments [32–36], where there is no energy/information exchange between the system and its environment.

In this work, we extend the study of the GP of open two-level system to the situation where there has an energy/information exchange between the system and its environment. The environment is at zero-temperature

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and has a Lorentzian spectral density, which corresponds to the radiation field as an environment being confined in a leaky cavity. We mainly concentrate on how the non-Markvoain effect affects the GP in different parameter regimes of the spectral density, and how the GP manifests its robustness against the decoherence.

This paper is organized as follows. In Sec. II, we introduce the model of a qubit interacting with a vacuum environment and analyze its decoherence behavior. In Sec. III we evaluate the GP of the qubit via performing analytical and numerical calculations. The correction effect exerted by the environment on the GP in different parameter regimes of the Lorentzian spectral density is analyzed. Finally, a brief discussion and summary are given in Sec. IV.

II. THE MODEL

Let us consider a system consisted of a two-level atom (qubit) coupled to a radiation field at zero temperature as an environment. The Hamiltonian of the system is [37]

$$H=\omega_0\sigma_+\sigma_-+\sum_k\omega_k a_k^\dagger a_k+\sum_k(g_k\sigma_+a_k+h.c.), \quad (2)$$

where $\sigma_\pm$ and ω_0 are the inversion operators and transition frequency of the qubit, $a_k^\dagger$ and a_k are the creation and annihilation operators of the k-th mode with frequency ω_k of the radiation field, and g_k is the coupling strength between the qubit and reservoir. Throughout this paper we assume $\hbar=1$. The system models the decoherence process of the atom via the amplitude decaying under the Born-Markovian approximation, which results in the spontaneous emission of the two-level atom [37] in quantum optics. The model is exactly solvable. The decoherence dynamics of the qubit is governed by the master equation [38]

$$\begin{aligned}\dot\rho(t) &= -i\Delta(t)[\sigma_+\sigma_-,\rho(t)]+\Gamma(t)[2\sigma_-\rho(t)\sigma_+ \\ &\quad -\sigma_+\sigma_-\rho(t)-\rho(t)\sigma_+\sigma_-], \end{aligned} \quad (3)$$

where the time-dependent parameters are given by

$$\Delta(t)=-\mathrm{Im}[\frac{\dot c(t)}{c(t)}],\ \Gamma(t)=-\mathrm{Re}[\frac{\dot c(t)}{c(t)}]. \quad (4)$$

It is shown that $c(t)$ satisfies

$$\dot c(t)+i\omega_0 c(t)+\int_0^t f(t-\tau)c(\tau)d\tau=0, \quad (5)$$

where $f(t-\tau)=\int J(\omega)e^{-i\omega(t-\tau)}d\omega$ is the environmental correlation function with the spectral density defined as $J(\omega)=\sum_k|g_k|^2\delta(\omega-\omega_k)$ and the initial condition $c(0)=1$. The time-dependent parameters $\Delta(t)$ and $\Gamma(t)$ play the roles of Lamb shifted frequency and decay rate of the qubit, respectively. The integro-differential equation (5) contains the memory effect of the reservoir registered in the time-nonlocal kernel function and thus the dynamics of qubit displays non-Markovian effect. If the time-nonlocal kernel function is replaced by a time-local one, then Eq. (3) recovers the conventional master equation under Markovian approximation.

It is obvious that the memory effect registered in the kernel function $f(t-\tau)$ is essentially determined by the spectral density $J(\omega)$. In this work we explicitly consider that the spectral density has a Lorentzian form [39]

$$J(\omega)=\frac{1}{\pi}\frac{W^2\lambda}{(\omega_0-\omega)^2+\lambda^2}, \quad (6)$$

where W is the coupling constant between the qubit and the environment, and λ defines the spectral width of the coupling at the resonance point ω_0. The Lorentzian spectral density describes that the vacuum radiation field as the environment is confined in a leaky cavity. Due to the leakage of the cavity field induced by the imperfection of the cavity mirrors, the spectrum of the cavity field displays a broadening at the resonance point regarding the atomic transition frequency ω_0. In this case one can verify that the correlation function decays exponentially $f(t-\tau)=W^2e^{-\lambda(t-\tau)}$ [39], which means that the parameter λ characterizes the correlation time of the environment as $\tau_c=\lambda^{-1}$. If τ_c is comparable with the typical time scale of the system, i.e. $\tau_0=1/\omega_0$, then the memory effect of the environment should not be neglected and the decoherence dynamics in this situation is non-Markovian. While $\tau_c\ll\tau_0$, the memory effect of the environment is negligible and the decoherence dynamics is Markovian. In the ideal cavity limit $\lambda\to0$, we have $\lim_{\lambda\to0}J(\omega)=W^2\delta(\omega-\omega_0)$, which corresponds to a constant kernel $f(t-\tau)=W^2$. Then the system reduces to the Jaynes-Cummings model with a vacuum Rabi frequency $g=W$.

Going back to the general case, one can obtain the analytical form of $c(t)$ by substituting the exponentially decaying $f(t-\tau)$ into Eq. (5) as

$$c(t)=e^{-\frac{(\lambda+i\omega_0)t}{2}}\left[\cosh(\frac{\Omega t}{2})+\frac{\lambda}{\Omega}\sinh(\frac{\Omega t}{2})\right], \quad (7)$$

where $\Omega=\sqrt{\lambda^2-4W^2}$. So the parameters in the master equation can be calculated readily

$$\Gamma(t)=\frac{2W^2}{\lambda+\Omega\coth(\Omega t/2)},\quad \Delta(t)=\omega_0. \quad (8)$$

One can see from Eqs. (8) when λ is much larger than other frequency scale, the decay rate tends to a constant value $\Gamma_0\equiv2W^2/\lambda$, which just characterizes the decoherence behavior of the qubit under the Markovian dynamics.

III. GEOMETRIC PHASE CORRECTED BY THE ENVIRONMENT

A. Analytical analysis

In the following we compute explicitly the GP of the qubit. We assume that the initial state of the qubit is chosen as

$$|\psi(0)\rangle = \cos\frac{\theta_0}{2}\,|+\rangle + \sin\frac{\theta_0}{2}\,|-\rangle\,, \tag{9}$$

where $|+\rangle$ and $|-\rangle$ are the excited and ground states of the qubit, respectively. This state corresponds to a vector in Bloch sphere with polar angle θ_0. The time-dependent reduced density matrix of the qubit under the initial condition (9) can be obtained straightforwardly from the master equation (3)

$$\rho(t) = \begin{pmatrix} \cos^2\frac{\theta_0}{2}|c(t)|^2 & \frac{\sin\theta_0}{2}c(t) \\ \frac{\sin\theta_0}{2}c^*(t) & 1-\cos^2\frac{\theta_0}{2}|c(t)|^2 \end{pmatrix}. \tag{10}$$

It is noted that besides the off-diagonal elements, the diagonal elements of the reduced density matrix $\rho(t)$ also change with time in our model. It is just this time-dependence of the diagonal elements of $\rho(t)$ characterizing the energy exchange between the qubit and its environment that makes our system shows dramatic difference to the dephasing model [32–36]. To calculate the GP of the qubit, we must firstly get the eigensolution of the reduced density matrix (10). The eigenvalues of the above reduced density are readily calculated,

$$\varepsilon_\pm(t) = \frac{1}{2}\left[1 \pm \sqrt{|c(t)|^2\sin^2\theta_0 + \left(2|c(t)|^2\cos^2\frac{\theta_0}{2} - 1\right)^2}\,\right], \tag{11}$$

It is obvious that the eigenvalue $\varepsilon_-(0) = 0$, which, from Eq. (1), means that the component of the eigenstate corresponding to the eigenvalue ε_- gives no contribution to the GP. Thus we only need to consider the eigenstate corresponding to the eigenvalue ε_+

$$|\varepsilon_+(t)\rangle = e^{-i\omega_0 t}\cos\Theta\,|+\rangle + \sin\Theta\,|-\rangle\,, \tag{12}$$

where

$$\cos\Theta = \frac{2\left(|c(t)|^2\cos^2\frac{\theta_0}{2} - \varepsilon_-\right)}{\sqrt{|c(t)|^2\sin^2\theta_0 + 4\left(|c(t)|^2\cos^2\frac{\theta_0}{2} - \varepsilon_-\right)^2}}. \tag{13}$$

Below we calculate the GP. Eq. (8) shows that the frequency shift of the qubit is zero for the Lorentzian spectral density [38]. So the period of the environment disturbed atom is the same as the one for a bare atom. Then the GP of the qubit acquired after a period $T = 2\pi/\omega_0$ can be calculated as

$$\Phi_g = \int_0^T \omega_0\cos^2\Theta dt. \tag{14}$$

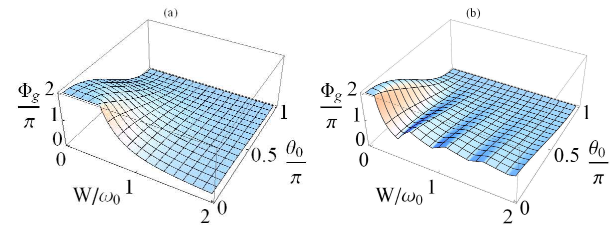

FIG. 1: (Color online) Numerical result of the GP as a function of the coupling constant and the initial angle. (a) Markovian dynamics with $\lambda = 5.0\omega_0$ and (b) non-Markovian dynamics with $\lambda = 0.05\omega_0$.

Here we point out that the expression of the GP (14) has been obtained by using the kinematic approach [20]. However, it can be shown that the expression is exactly the same as the one obtained by the definition of Pantcharatnam's phase of a nonunit vector ray [22], which exhibits some essential features of the GP in such systems.

From Eq. (14), one can notes that the effect of the environment enters into the GP solely via the time-dependent factor $|c(t)|^2$ of the excited-state population. When the environment is absent, then $|c(t)|^2 = 1$ and the GP reduces to $\Phi_g^{(0)} \equiv \pi(1+\cos\theta_0)$, which is just the GP acquired by a two-level atom in the unitary dynamics. Depending on the decoherence dynamics being Markovian or non-Markovian, the short-time dynamics of $|c(t)|^2$ shows remarkably different behaviors. In the Markovian dynamics, $|c(t)|^2$ decays monotonically and finally approaches zero. In this case, the larger the decay rate Γ_0 is, the larger the correction of the GP should be. In the non-Markovian dynamics, contributed from the memory effect of the environment $|c(t)|^2$ shows transient oscillation with time, which naturally induces a correction of the GP different to the one in Markovian dynamics, as shown in the following.

Up to the first order of coupling strength W^2, i.e. the weak coupling limit, we have

$$\Phi_g^{(1)} = \Phi_g^{(0)} - \left(\frac{W}{\omega_0}\right)^2 \sin^2\theta_0\left(1+\frac{\cos\theta_0}{2}\right) z\left(\frac{\lambda}{\omega_0}\right), \tag{15}$$

where $z(x) \equiv x^{-3}\left[1 - e^{-2\pi x} - 2\pi x(1-\pi x)\right]$. Besides the leading term $\Phi_g^{(0)}$ corresponding to the well-known GP acquired under the unitary dynamics, the second term, which is quadratic in W/ω_0 with a λ-dependent coefficient $z(\lambda/\omega_0)$, is the lowest-order correction to the GP induced by non-unitary dynamics due to the interaction with the environment. It is easy to check that $z(\lambda/\omega_0)$ is a monotonically decreasing function with the increase of λ, so we can expect that the GP shows a larger deviation to $\Phi_g^{(0)}$ for a small λ than a large one. In particular, in the ideal cavity limit $\lambda \to 0$, the function $z(\lambda/\omega_0)$ arrives at its maximum $4\pi^3/3$, where the GP has a largest correction in this weak coupling (or small W)

regime. On the other hand, when $\lambda \gg \omega_0$, W, one can verify $z(\lambda/\omega_0) \to 2\pi^2\omega_0/\lambda$. Consequently, the GP in this Markovian limit is

$$\Phi_g^{(1)} = \Phi_g^{(0)} - \frac{\pi^2\Gamma}{\omega_0}\sin^2\theta_0\left(1+\frac{\cos\theta_0}{2}\right), \qquad (16)$$

which shows very similar form to the result of [26] obtained from the Born-Markovian master equation.

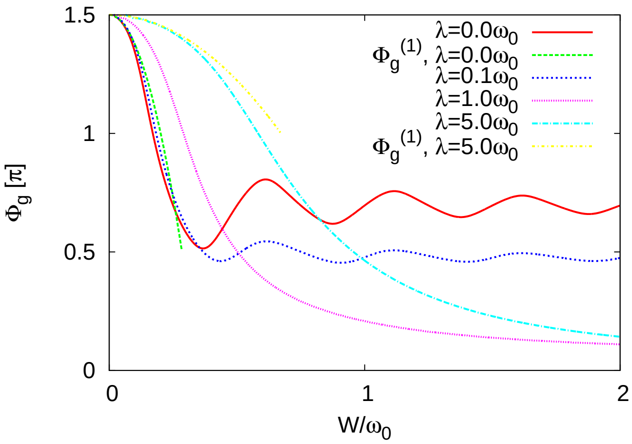

FIG. 2: (Color online) The exact GP and $\Phi_g^{(1)}$ as a function of the coupling constant under different spectral width λ. The initial polar angle is taken as $\theta_0 = \pi/3$.

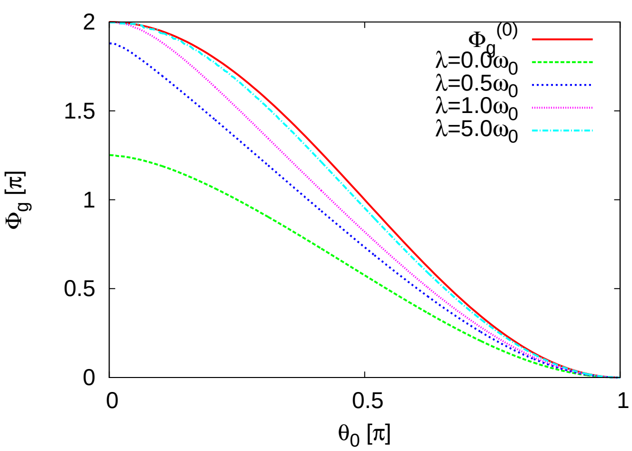

FIG. 3: (Color online) The exact GP and $\Phi_g^{(0)}$ as a function of the initial polar angle under different spectral width λ. The coupling constant is $W = 0.2\omega_0$.

B. Numerical results

In the following we give the exact GP via numerically evaluating the integration in Eq. (14). In Fig. 1 we plot the exact GP as a function of the coupling constant and the initial polar angle. When $\lambda = 5.0\omega_0$, the correlation time of the environment $\tau_c = 0.2/\omega_0$ is much less than the typical time scale of the system τ_0. In this situation, the memory effect of the environment is negligible and the dynamics of the system is Markovian. Consequently, the energy and the information flow single-directionally from the system to the environment, which results in the dissipation to the system. The dissipation time scale in this Markovian dynamics is characterized by $\tau_D = 1/\Gamma_0 = \lambda/2W^2$. We can see in Fig. 1(a) that the GP in this situation decreases monotonically with the increase of the coupling constant. This is understandable that a larger coupling constant would induce a stronger dissipation and a short decoherence time scale τ_D to the qubit system. With the increase of W, τ_D approaches T, which means that the dissipation becomes more and more notable within the time scale T. On the other hand, when W is small the GP shows small deviation to the unitary one $\Phi_g^{(0)}$. In this situation, τ_D is larger than T, making the dissipation negligible within the time scale T during which the GP is accumulated. This interesting phenomenon just reflects the resilient ability of the GP to the environment's disturbance, especially in weak coupling limit. A similar phenomenon is also observed in the dephasing model [36]. When $\lambda = 0.05\omega_0$, $\tau_c \gg \tau_0$ and the non-Markovian effect induced by the memory effect of the environment to the system would show its distinct impact on the the dynamics of the system. Besides the dissipation, the environment also exerts a dynamical backaction on the system [40]. The backaction effect is reflected in that the energy and the information flow back and forth between the system and the environment. In this situation we can see in Fig. 1(b) that the GP oscillates with the increase of W and then tends to a definite value, which is qualitatively different to the above Markovian situation. The oscillation is just the manifestation of the dynamical backaction. It is very clear that the non-zero character of the GP in the case of large W is due to the counteracted competition between the backaction and dissipation exerted by the non-Markovian environment partially, which weakens the decoherence of the qubit system.

A more clear comparison of the GP with different λ as a function of W when $\theta_0 = \pi/3$ is shown in Fig. 2. As comparisons, the perturbed results for $\lambda = 0$ and $5.0\omega_0$ are also presented in this figure and they agree well with the corresponding exact one only in the weak coupling limit. We can see obviously that in the weak coupling regime the environment with a large λ induces a smaller correction to the GP than the one with a small λ. That is, the non-Markovian effect has a strong correction to the GP in this weak coupling regime. This is consistent with the result obtained in Ref. [41], where a phenomenological analysis of the non-Markovian effect on the GP is performed. While in the strong coupling regime the situation is opposite, because the system with a large-λ shows stronger decoherence than the one with a small λ where the non-Markovian effect has a strong correction

to the GP.

Even till today it is still difficult to access experimentally the strong coupling regime in the cavity QED platform. For example, in a recent microtoroidal resonator experiment [42], the achieved coupling strength between the atom and the cavity field is 90MHz, while the damping rate of the cavity field is about 180MHz. Such a large damping rate would induce a noticeable spectrum expansion of the cavity mode. It is of course interesting to examine the GP in this bad-cavity and weak coupling regime. In Fig. 3 we plot the GP as a function of the initial polar angle in the weak coupling regime for different spectrum width λ. One can see that the GP approach more and more closely the unitary GP $\Phi_g^{(0)}$ with the increase of λ. This accounts for once again that the GP shows more strong resilient ability to the noise of the Markovian environment with a large λ. Since this resilient character is obtained in weak coupling and bad-cavity condition, we argue that it is accessible by modern cavity QED experiment [42].

IV. SUMMARY AND DISCUSSION

We have studied exactly the GP for a qubit in an amplitude decaying environment using the kinematic approach. We have evaluated the non-Markovian effect on the GP. It is demonstrated that the non-Markovian environment has a more significant correction to the GP in the weak coupling regime due to the strong short-time correlation between the qubit and the environment. Interestingly, our result also indicates the insensitivity of the GP to the Lorentzian environment in the Markovian regime when the coupling is weak. It just manifests the resilient ability of the GP to the environmental noise. This results elucidate that the GP in this cavity QED system is fault-tolerant not only against the classical noise induced by the parameter fluctuation [11, 15], but also against the quantum noise. This has significant meaning in geometric quantum information processing.

Our model, as a basic model of quantum optics, is particularly relevant to the cavity QED experiments. In this respect some remarkable experiments, such as the efficient coupling of the trapped atoms with cavity field [43, 44], have been performed successfully. Practically, any phase variation is observed only via some kind of interferometry between the involved state and certain selected reference state. For example, the GP for mixed state has been observed via designing a quantum network in NMR system to realize the interferometry [23, 24]. This provides a clue to observe GP in cavity QED system. Although the GP has not been observed in the cavity QED system, a quantum network using the recent developed microtoroidal resonator [42] has been proposed [45] based on the input-output process of photons [46]. If an effective interferometry could be realized in this quantum network, then the GP would be expected to be observable in cavity QED system. Our work on the assessment of the environmental effect on the GP, especially in non-Markovian regime, is of great importance in using the GP in cavity QED system to implement the quantum gates.

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